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organized by

Ardeshir Guran

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Claudia Timofte (ROMANIA)
Katrin Laas (ESTONIA)
Kamran Iqbal (USA)
Oleg Kirillov (Germany)
Nikolai Magnitskii (Russia)
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On the Asymptotic Behavior of a Reaction-Diffusion System in a Porous Medium

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The effective behavior of a reaction-diffusion system in a porous medium is analyzed. We deal, at the microscale, with an ε-periodic structure Ω, consisting of two parts: a fluid phase Ω^ε and a solid one, $\Omega \setminus \overline{\Omega}^\varepsilon$; ε represents a small parameter related to the characteristic size of the solid grains.

In such a domain, we analyze the asymptotic behavior, as $\varepsilon \to 0$, of a coupled system of equations, involving diffusion, adsorption and non-smooth chemical reactions. Assuming that the surface of the solid part is physically and chemically heterogeneous and allowing also a surface diffusion modelled by a Laplace-Beltrami operator to take place on this surface, we prove that the effective behavior of our system is governed by a new boundary-value problem, with an additional microvariable and a zero-order extra term proving that memory effects are present in this limit model.

Key words: homogenization, reactive flows, adsorption, Laplace-Beltrami operator.

Resonant Behavior of a Fractional Oscillator with Random Damping Micro-channel

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As the fractional oscillator (FO) is the simplest toy model for different phenomena in viscoelastic media it is a typical theoretician's paradigm for various fundamental ideas in the fields of science and engineering. We investigate the behavior of a FO in the case of a fluctuating damping coefficient. Although the behavior of the FO with a random eigenfrequency has been investigated in some detail [1], it seems that analysis of the potential consequences of interplay between a random damping and memory effects incorporated in the FO is absent in literature. This is quite unjustified in view of the fact that the importance of multiplicative fluctuations of damping for various natural systems, *e. g.*, for water waves influenced by a turbulent wind field, has been well recognized [2]. Thus motivated, we consider a FO with a power-law memory kernel subjected to an external periodic force. The influence of the fluctuating environment is modeled by a multiplicative dichotomous noise (fluctuating damping) and an additive noise. The main purpose of this work is to provide exact formulas for the analytic treatment of the dependence of spectral amplification on the system parameters: viz. the noise correlation time, noise amplitude, memory exponent, and driving frequency. Based on those exact expressions we demonstrate that stochastic resonance is manifested in the dependence of the spectral amplification upon the noise parameters. Furthermore, we will show that in certain parameter regions the FO exhibits a multiresonance behavior versus the driving frequency. Moreover, a couple of critical memory exponents are found which mark the transitions between various dynamical regimes of the oscillator.

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Postural Stability Analysis and PID Controller Synthesis for Movement Coordination in a Multi-Segment Biomechanical Model

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Biomechanics of the human body are characterized by an abundance of degrees of freedom that must be controlled and managed to realize a coordinated movement. Quiet standing involves incessant postural adjustments aimed to counter the multidimensional disturbances to standing posture. Motor control of postural and voluntary movements is executed as a series of motor programs that specify muscle synergy, sequencing of contraction, relative timings, and durations. The sensorimotor cortex translates these programs into mechanical stiffness at the joints, movement direction, velocity and end points. The spinal cord helps set the muscle tone and stiffness, and the sensitivity of primary sensory organs, the muscle spindle (MS) and the golgi tendon organ (GTO).

Postural stability constitutes an important attribute of the musculoskeletal-proprioceptive apparatus. Postural stability is enabled by the biological servomechanism involving muscles, stretch receptors and neural pathways. The servo system generates reflexes aimed at relieving muscle tension, and returning muscle tone to its preset level. Passive viscoelasticity of the musculo-tendon complexes (MTCs) is necessary but not sufficient for stability. Stability augmentation is delivered through persistent central nervous system (CNS) involvement via spinal reflexes, internal model control, and anticipatory postural adjustments (APA). Active control of muscle stiffness and tone relies on proprioceptive feedback to dynamically modulate muscle afferents and maintain balance. However, intrinsic delays in the reflex pathways and the low-pass characteristics of the muscle response tend to limit the effectiveness of active mechanisms of postural stabilization. How then standing humans are able to maintain balance remains an open and intriguing question.

The use of mathematical models to provide insight into neurophysiology has a long history. Control of balance in human upright standing is particularly well suited for modeling, and is also a popular experimental paradigm. In this study we consider a simplified characterization of the postural control system that embraces two broad components: one representing the musculoskeletal dynamics in the sagittal plane, and the other representing proprioceptive feedback and the decision making modalities of the CNS. Specifically, a four-segment sagittal model of the physiological system is developed that includes important physiological constructs such as Hill-type muscle model, active and passive muscle stiffness, force feedback from the GTO, muscle length and rate feedback from the MS, and transmission latencies in the neural pathways. A proportional-integral-derivative (PID) controller for each individual degree of freedom (DOF) is assumed to represent the CNS analogue in the modeling paradigm. The overall control structure for postural stability and movement coordination consists of three components: 1) a reference trajectory; 2) a set of autonomous PID controllers; and, 3) the position, velocity, and force feedback loops with physiological latencies.

We present analytical and simulation results to show that the proposed representation adequately shapes a postural control that: a) possesses good disturbance rejection and trajectory tracking, b) is robust against feedback latencies and torque perturbations, and c) is flexible to embrace changes in the musculoskeletal parameters. We additionally present detailed sensitivity analysis to show that control under conditions of limited or no proprioceptive feedback results in: a) significant reduction in the stability margins, b) substantial decrease in the available stabilizing parameter set, and c) oscillatory movement trajectories. Overall, these results suggest that anatomical arrangement, active muscle stiffness, force feedback, and physiological latencies play a major role in shaping motor control processes in humans.

Paradoxes of Dissipation-Induced Instabilities in Mechanics and Physics

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In 1952 Ziegler found that an infinitesimally small amount of damping leads to a finite change in the stability domain of a two-link pendulum loaded by the follower force. In 1956 Bottema resolved this destabilization paradox by means of the Whitney umbrella singularity that as he established exists on the stability boundary of the damped Ziegler's pendulum. I will talk about extensions of this result to general finite dimensional and continuous circulatory systems as well as to the gyroscopic systems with small damping and non-conservative positional forces.

Examples of similar paradoxal phenomena from rotor dynamics, continuum mechanics and magnetohydrodynamics will be considered in detail. A broad overview of the achievements in this field over the last half a century will be given.

THE UNIVERSAL THEORY OF CHAOTIC DYNAMICS WITH APPLICATIONS IN NONLINEAR DIFFERENTIAL EQUATIONS AND TURBULENCE IN FLUIDS

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Abstract

The universal FSM (Feigenbaum-Sharkovskii-Magnitskii) - theory of transition to dynamical and spatio-temporal chaos in autonomous and non-autonomous, dissipative and conservative, ordinary and partial nonlinear systems of differential equations and differential equations with delay arguments through the subharmonic and homoclinic cascades of bifurcations of stable cycles or stable tori is presented. All propositions are illustrated by numerous examples of all kinds of nonlinear differential equations and, in particular, examples of FSM-scenarios of transition from laminar to turbulent regimes in viscous incompressible 3D fluid motion in Rayleigh-Benard convection and in a motion of a fluid behind a ledge. The problem of turbulence is named by Clay Mathematics Institute as one of seven millennium mathematical problems and it is also in the list of 18 most significant mathematical problems of XXI century formulated by S.Smale.

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Computational Approaches to the Lorenz System

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Abstract:

For almost 50 years, the Lorenz attractor [1], with its intriguing double-lobed shape and chaotic behavior, has symbolized order within chaos in dynamical systems. The Lorenz attractor dates from 1963, when the meteorologist Edward Lorenz published an analysis of a simple system of three differential equations that he had extracted from a model of atmospheric convection. He pointed out that they possess some surprising features. In particular, the equations are 'sensitive to initial conditions', meaning that tiny differences at the start become amplified exponentially as time passes. Since then, the Lorenz System has been the subject of many articles [2, 3, 4], monographs [5], textbooks [6, 7], and university theses [8, 9].

In this paper we first present a new Differential Quadrature Formulation for the solution to the Lorenz System with and without a white noise. The results are compared to those obtained by Runge-Kutta algorithm. It is concluded that the DQ gives a quicker convergent solution for both chaotic and non-chaotic response.

Then, the Karhunen-Loeve (K-L) basis and the Wiener-Hermite (W-H) kernel functions [10] are evaluated and the relevant closures applied. The accuracy of the results was compared with the numerical results obtained in previous part.

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Energy Transfer in Ratchets Driven by Additive Trichotomous Noise

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The current boom of the ratchet effect, *i.e.*, a directed motion of Brownian particles induced by nonequilibrium fluctuations, with no macroscopic driving applied, in spatially periodic structures has started with Magnasco‘s theoretical work [1]. The initial motivation in this field has come from biology, in particular from the studies of the mechanism of vesicle transport inside eukaryotic cells. Beyond that it was suggested that the ratchet mechanism can be used for obtaining efficient separation methods of nanoscale objects. In our work, the efficiency of the energy transformation of overdamped Brownian particles in a tilted periodic sawtooth potential driven by a nonequilibrium three-level noise and an additive thermal noise is considered analytically. All the physical results discussed have been computed by means of exact formulas. It is established that in a certain parameter region the dependence of the efficiency of energy transformation on noise parameters exhibits a bell-shaped form. Thus, in such parameter regions an increase of the values of noise characteristics (temperature, noise-flatness, correlation time, and noise amplitude) can facilitate the conversion of noise energy into mechanical work. The connection of such a resonance-like behavior of efficiency with the phenomenon of multiple current reversals [2] is also discussed.

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The Static and Dynamic Stability of Carbon Nanotubes

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This lecture reviews recent research studies on the stability of carbon nanotubes. We first introduce the audience to the structure and properties of carbon nanotubes. Then, various instabilities exhibited by carbon nanotubes are presented. In the second part of this lecture we introduce the two most used methods for stability analysis of carbon nanotubes, *i.e.*, continuum models and atomistic simulations. We close the talk with delineating the main factors, such as dimensions, temperature, strain rate, and boundary conditions, influencing the stability of carbon nanotubes. It is hoped that this work provides current knowledge on the stability of carbon nanotubes, reviews the analytical as well as computational methods for determining the margins of stability, and inspires researchers to further investigate the instabilities of carbon nanotubes for better design and practical applications.