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BOOK OF ABSTRACTS

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Weighted Semigroup Algebras as Dual Banach Algebras

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In this paper, among other things, we study those conditions under which the weighted semigroup algebra $l_1(S, \omega)$ is a dual Banach algebra with respect to predual $c_0(S)$. Some useful examples, illustrating the results, are also included.

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Explicit High Resolution Finite Difference Schemes for a Juvenile-Adult Amphibian Model

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We consider a juvenile-adult model for an amphibian population in which juveniles are structured by age and adults are structured by size. We develop first and second order explicit schemes to approximate the solution of the model. Stability and convergence of both schemes are proved. Numerical examples demonstrating the high resolution property and the achievement of the designed accuracy are provided. The schemes are then applied to understand the dynamics of an urban Green Treefrog population.

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Planes and Axes of Symmetry in a Piezoelectric Material

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Properties of a piezoelectric material are described by means of a tensor, e_{ijk}, which has rank 3 and is symmetric with respect to the last two indices, *i.e.*, $e_{ijk} = e_{ikj}$. Because of this symmetry, the tensor has 18 independent components. It is convenient to represent the tensor in the form of the following 3×6 matrix.

$$\mathbf{e} = \begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} \\ e_{21} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} \\ e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & e_{36} \end{bmatrix}$$

where the familiar two-index notation is used. Necessary and sufficient conditions are derived for a piezoelectric tensor to belong to a symmetry group characterized by a plane of symmetry. If $\mathbf{n}$ and $\mathbf{m}$ are unit vectors orthogonal to each other, it is necessary and sufficient for $\mathbf{n}$ to be a normal to a plane of symmetry of a piezoelectric material that

(a) it is orthogonal to each of the vectors $\mathbf{E}_1 = e_{kjj}$, $\mathbf{E}_2 = e_{jjk}$, $\mathbf{E}_3(\mathbf{n}) = e_{ijk}n_i n_j$, and

(b) it is parallel to each of the vectors $\mathbf{F}_1(\mathbf{n}, \mathbf{m}) = e_{ijk}n_i m_j$, $\mathbf{F}_2(\mathbf{n}, \mathbf{m}) = e_{ijk}m_i n_j$, $\mathbf{F}_3 = e_{kij}m_i n_j$.

A similar result holds for $\mathbf{n}$ to be an axis of symmetry.

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Image Reconstruction in Acoustic Reflectivity Tomography

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In various applications of acoustic reflection tomography the support of the image function lies outside of the data acquisition set, which is usually a closed curve in 2D or a closed surface in 3D. The problem of image reconstruction in this setup is highly unstable, yet necessary and important (*e.g.*, in intravascular ultrasound). Although one can not expect stable reconstruction of the whole image, microlocal considerations show that certain image singularities can be reconstructed correctly. There are some uniqueness results and exact inversion formulas for a few restricted cases of this exterior problem, however no robust inversion algorithm has been developed so far to recover the "visible" singularities. The talk will discuss the known results and recent advances in this direction.

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Some Problems of Observations' Control

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Problems of observations' control are considered for an object the state x of which is governed by a system of ordinary differential equations. There is a controller who controls the object by means of function v and an observer who can measure a signal $y_i = g(x(t_i)) + w_i$, where errors w_i belongs to a compact set W, $0 \leq t_1 < ... < t_N \leq T$. The controller knows about presence of the observer and, pursuing own aims, aspires to complicate the supervision process. Similar problems can be studied both in the determined statement, and in the stochastic-minimax statement. It is supposed that disturbances and system trajectories can be constrained by the additional conditions unknown to the observer who knows only the *a priori* restrictions on the specified values or on the moments of the noise. The observer interprets a part of entrance influences as the noise which does not have the suitable statistical description. However the mentioned values can have quite concrete physical sense and be used by the controller both for initiation of active hindrances to the observer, and for achievement of other purposes not known to him. Knowing the movement and observation equations, and also the sets V, W, X_0, where X_0 is a compact set of initial states, the observer builds the information sets X_i that is the intersection of the attainability domain $F_i(X_{i-1}, V)$ from the set X_{i-1} under constant controls on $[t_{i-1}, t_i]$ and the set $\{x : g(x) \in y_i - W\}$. The problem for the controller is to maximize over parameters v, w, x_0 the functional I which is the sum of functions $f_i(x(t_i), X_i)$ under an additional restriction $v \in V_i$, where V_i is a subset of V. The Hausdorf distance $h(x(t_i), X_i)$ or the value max $||x||$ over elements of information set can be used as functions f_i. The problem is solved via the dynamic programming. Work is supported by the RFBR, the grant No 10-01-00672a, and by the program of Presidium of RAS 'Mathematical theory of control', the project 09-P-1-1014

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Mathematical Modeling of Sterile Insect Technology for Control of Anopheles Mosquito

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Anopheles mosquito is a vector responsible for the transmission of diseases like Malaria which affect many people. Hence its control is a major prevention strategy. The Sterile Insect Technology (SIT) has been know for relatively long time but not yet used for mosquito control. With the demand for nonpolluting control methods SIT is receiving an increased attention. In this paper we design a mathematical model for the effect of SIT as a control measure on the Anopheles mosquito population. The aim is to design efficient strategies leading to reduction of the mosquito population below certain epidemiologically relevant threshold. The mathematical analysis of the model deals mainly with its properties as a dynamical system. Naturally, these include the equilibria and their stability. We also place an essential emphasis on establishing properties of global nature like dissipativity of the system, global asymptotic stability of an equilibrium when it is unique and the basins of attraction of stable equilibria if they are more. This analysis is based on an application of the theory of monotone dynamical systems.

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An Approach to the Numerical Integration of Age-structured Population Models with Finite Life Span

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We will carry out a brief historical revision of age-structured populations models. These models have evolved to consider populations with finite life span,

$$u_t + u_a = -\mu(a, I_\mu(t), t)\, u, \quad 0 < a < a_\dagger,\ t > 0,$$

$$u(0,t) = \int_0^{a_\dagger} \alpha(a, I_\alpha(t), t)\, u(a,t)\, da\ , \quad t > 0,$$

$$u(a,0) = \phi(a), \quad 0 \le a \le a_\dagger,$$

where

$$I_\mu(t) = \int_0^{a_\dagger} \gamma_\mu(a)\, u(a,t)\, da,\ I_\alpha(t) = \int_0^{a_\dagger} \gamma_\alpha(a)\, u(a,t)\, da\ ,\ t > 0.$$

This model has a great interest since, on one hand, in the study of human demography, the existence of very old ages that are not reached by any individual in the population is a empiric fact. On the other hand, it is the base of others used in diverse disciplines as ecology, cellular dynamics, *etc.*

Also, it will be carried out a revision of the more significant numerical schemes that have been considered in the literature for the integration of such age-structured population model and it will be commented briefly their main characteristics.

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Time Delay Models of Business Cycle

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Delay differential equations along with differential equations and difference equations are widely used by the modeling of economic phenomena [1-3]. In this lecture I will present the basic properties of the three nonlinear business cycle models with delays:

1) the Kaleckian models;

2) the delayed Kaldor-Kalecki model;

3) the Goodwin Model with fixed investment lag.

I will compare the effects caused by two different approaches: fixed time delays and continuously distributed time delays. In preparing the lecture used the results of [1-13].

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A Distributed Time Delay Goodwin Models of the Business Cycle

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A. Matsumoto, F. Szidarovszky [1] have compared the Goodwin model of business cycle [2] with fixed and continuously distributed time lags. They showed that for small delays the two types of models generate identical local asymptotic behavior; for large delays the asymptotic properties become different. Continuously distributed time delay Goodwin's model of the business cycle can be written in the form of system of integro-differencial equations,

$$\epsilon\dot{y}(t) + (1-\alpha)y(t) = \phi(\dot{y}_d(t)) + A(t), \qquad t > 0$$
$$y_d(t) = \int_{-\infty}^{0} k(t+s)y(-s)ds, \quad y(t) = \phi(t), \qquad -\infty \le t \le 0.$$

Here $y(t)$ is income, $\varepsilon > 0$ – the adjustment coefficient, α – the marginal propensity to consume, $0 \le \alpha \le 1$, ϕ – the nonlinear induced investment function, $A(t)$ – the autonomous investment, k – the delay kernel, $\int_0^\infty k(s)ds = 1$. In this talk we show that an important role plays not only the magnitude of delay, $\tau = \int_0^\infty sk(s)ds = 1$, but also the variance of the delay distribution $\sigma^2 = \int_0^\infty (s-\tau)^2 k(s)ds = 1$ (delay distribution width). If σ is chosen to be very small, $\sigma \ll \tau$ the time behavior of solutions is similar to fixed delay case described in [3, 4]. The distributed delays yield simpler behavior compared with the fixed delay case: oscillations amplitudes and periods may be different or may even be replaced by the damped oscillations.

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Some Properties of Integro-differential Equations from Biology

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We present some models of integro-differential equations from population dynamics, where the integral term describes the nonlocal consumption of resources. Fredholm property of the corresponding linear operators are useful to prove the existence of travelling wave solutions. For some models, this can be done only when the support of the integral is sufficiently small. In this case, the integro-differential operator is close to the differential one. One uses a perturbation method which combines the Fredholm property of the linearized operators and the implicit function theorem. For some other models, Leray-Schauder method can be applied. This implies the construction of a topological degree for the corresponding operators and the establishment of a priori estimates for the solution. Some biological interpretations follow from this study.

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An Analytical Study of the Stochastic Super-Exponential Growth Model for Human Population Dynamics

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According to the historical estimates of world population, human population has grown super-exponentially for most of the known history. Recent DNA sequencing analysis reveals that human population has grown super-exponentially for the last 10,000 years. Super-exponential growth model with environmental perturbations has been considered in this paper. Environmental noise has been modeled as Gaussian white noise interpreted a la Stratonovich. Model has been studied analytically and exact results have been presented. Exact formulae for conditional probability density, the mean value and the higher statistical moments of the population abundance have been calculated and rendered. Also conditional probability density, mean value of the lifetime and most probabilistic value of the occurrence of phase transition of the population, have been presented and analyzed. Noise induced behavior coupled with the internal nonlinearity of the dynamics within the system has been discussed.

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Numerical Method for 3D Simulation of Foam Dynamics in the Presence of Surfactant

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A 3D numerical method is presented for simulation of foam formation and dynamics in viscous flows in the presence of insoluble surfactant and under the influence of van der Waals forces. The mathematical model is based on the Stokes equations in the fluid phases, coupled with velocity and stress boundary conditions at the interfaces. A nonuniform surfactant concentration on the interfaces, governed by a convection-diffusion equation, leads to a gradient of the interfacial tension, which in turns leads to an additional tangential stress on the interfaces. The presented numerical method is a semi-implicit coupling of a boundary-integral method for the velocity in the fluid phases with a finite-volume method for the surfactant concentration on the interfaces. Additional elements of the method are: Nonsingular contour integration of the singular layer potential; Higher order approximation of the interface position; Dynamic mesh regularization.

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Bifurcation Analysis in a Model of 1,2-dichloroethane Biodegradation by Klebsiella Oxytoca va 8391 Immobilized on Granulated Activated Carbon Using the Maple Packages BifTools and IntervalTools

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Biotechnological processes are modeled by systems of autonomous ordinary differential equations depending on parameters. Given such a system, the ultimate goal is to obtain its bifurcation diagram, that is to divide the parameter space into regions within which the system has topologically equivalent phase portraits. A natural way for doing that is to use the center manifold theory and normal form theory [2]. In many cases however especially in more complicated models to use these two theories leads to lengthy intermediate calculations and expressions. The computer algebra systems and their tools for bifurcation analysis can help us in these cases. The Maple packages BifTools and IntervalTools [3] are such kind of tools and in this paper we apply them to investigate a mathematical model of the biodegradation of xenobiotics by microbial cells attached to particles of granulated activated carbon [1].

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Hierarchical Classification and Functional Data Analysis Techniques

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In this paper, we propose an algorithm for hierarchical classification, based on an ultrametric distance. We study its properties and develop an application in Microsoft Visual Studio,based on the algorithm proposed, using C language. The software obtained will be used to classify the shares from Bucharest Stock Exchange which had profit during the last two years, in order to find similarities and differences between these shares and build a diversified portfolio. We prove that this portfolio is representative for the shares from Bucharest Stock Exchange and study the evolution of the obtained portfolio at different moments of time, using functional data analysis methods (STATIS).

In order to evaluate our methodology, we provide a numerical experiment .We demonstrate the performance of the proposed algorithm by comparing the obtained results with the evolution of BET index, BET-C index or BET -XT index, which are representatives for the capital market in Romania.

Keywords: classification, algorithm, software, analysis methods.

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The Matrix: Mathematic Modeling for the Liberal Arts: Graphic Literacy

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Mathematics, the language in which science came to exist, now drives the structure, design and process of the liberal arts education. Mathematics, brought to full power through technology, has morphed into new expressions and new abilities. It is now a laboratory discipline that allows deeper investigation, better understanding and more effective participation in Liberal Arts fields. Mathematics is the new microscope for research and analysis; mathematic modeling is the new form of communication for discovery and insight. Liberal Arts students are challenged to help solve increasingly complex problems from their own professional perspective; mathematics in the form of computer graphic literacy massively accelerates student experience, and will greatly increase the skills of analysis and intuition.

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The Helmholtz Equation in 270 Degrees Wedge Sectors Facing Dirichlet, Neumann and Impedance Boundary Conditions

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We will consider boundary value problems for the Helmholtz equation in the exterior of a quadrant in a Bessel potential spaces framework. The problems will be formulated by using the Helmholtz equation subjected to different possible combinations of boundary conditions on the faces of the corresponding wedge. Namely, under consideration there will be boundary conditions of Dirichlet-Dirichlet, Neumann-Neumann, Neumann-Dirichlet, impedance-Dirichlet, and impedance-Neu-mann types. Existence and uniqueness results are proved for all these cases in the weak formulation. In addition, the solutions are provided within the spaces in consideration, and higher regularity of solutions is also obtained in a scale of Bessel potential spaces.

The talk is based on a joint work with D. Kapanadze.

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Soliton Interactions on a Boussinesq type Equation with a Linear Restoring Force

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The equation governing the flexural deformation of an elastic rod on elastic foundation (EREFE) is only of fourth order, but because of the presence of the linear restoring force, it exhibits some of the phenomenology of the 6GBE, especially when the nonmonotonic shapes are concerned. Yet, the linear term presenting the restoring force, changes qualitatively the equation for the stationary waves. Here, we will treat the problem numerically, initially considering it in the moving frame, stationary problem, and then, using the solutions obtained earlier as initial condition, extent the investigation to the time dependant problem where we study the propagation and interaction of the solitary waves.

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On the Pseudolocalized Solutions in Multi-dimension as Candidates for Quasi-Particles

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A new class of solutions of three- and higher- dimensional equations from Boussinesq paradigm are considered, whose profiles are not localized functions in the sense of the integrability of the square over infinite domain. For the new type of solution, the gradient and/or the Hessian/Laplacian are square integrable. In the linear limiting case, analytical expressions for the profiles of the pseudolocalized solutions are found and the method of variational approximation is applied to find the dynamics of the centers of the quasi-particles (QPs) based on these solutions. The discrete Lagrangian is derived for the case of propagation of the QP is derived and the pseudomass of a QP is defined. At long distances attraction force proportional to the inverse square of the distance (in the 3D case) is found, which is clear analog to the gravitation force.

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Models of Antibody Responses to Viral Infections

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During the course of an individual's infection with Human Immunodeficiency Virus (HIV), the virus population consists of a distribution of different variants, produced by mutation and selection. Consequently, the immune system attempts to build a response that is broad enough to handle the diversity of virus strains present. Biological experiments have shown that neutralizing antibodies fail to offer long-term protection because they are primarily strain-specific and lag behind viral evolution. We develop mathematical models of antibody mediated immune responses against HIV with an emphasis on their neutralizing activity. Analysis of the models help us predict which factors (host or virus specific) influence the outcome of the infection. In particular, we determine the roles of competition and cross-reactivity between families of neutralizing antibodies in the presence and absence of virus evolution, as well as the need to neutralize more than one viral spike to ensure protection.

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The Quasi Lorentz Transformation for Rotating Objects

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The quasi Lorentz transformation introduced recently, is a differential first order v/c approximation to the Lorentz transformation, providing a tool for analyzing electromagnetic scattering problems involving non-uniform motion. Using the so-called slip-shells model, the QLT is applied here to systems of reference rotating with respect to the initial frame (aka laboratory frame), in which the incident wave is launched. It is shown that an observer rotating relative to a monochromatic plane wave will measure a Bessel series type frequency spectrum. The problem of scattering by rotating circular cylinders is analyzed. For axial motion, the scattered wave in the initial frame shows no Doppler frequency shifts, as expected. However, for material cylinders, the scattering amplitude (radiation pattern) and the associated scattering coefficients show velocity effects. Moreover, these effects depend on the sense of rotation. Due to the frequency spectrum in the cylinder's rest frame, dispersion effects will be displayed. It is noted that unlike the instantaneous velocity approximation model, based on the Minkowski constitutive relation, the present model has the potential of dealing with non axially symmetric rotating scatterers. As an example for non-axial motion, the problem of orbiting cylinders is analyzed. In this case Doppler frequency shifts are evidenced, as expected.

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Albert's Cow and Special Relativity: A Layman's Introduction (Almost) without Mathematics

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This is a popular introduction to SR, for laymen motivated by being exposed to too many bogus TV programs about the wonders of modern physics, without any attempt to explain any of these wonders. The presentation does not go beyond high school junior class physics and mechanics, and mathematics involving two simple equations with two unknowns. The motivation comes from listening to popular talks about various subjects, and the constant questioning: "and what were you doing before you retired?" At this stage I usually start to wave my arms energetically, and comment how radio waves are important to cellular telephony.

Many images and some explanations found on the internet are used. Trying to get approval from all these sources would take years. So here is a collective "thank you," and an apology for not giving detailed credit, but this humble presentation is not for publication anyhow.

The explanations are honest, although not all the details are meticulously explained. So there are here lies of the kind "suppression veri" but not "suggestio falsi." My apologies.

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Stationary Cylindrical Couette Flow at Different Temperature of Cylinders. Local Knudsen Number Effect

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The stationary Couette gas flow between rotating inner cylinder and stationary outer is considered using DSMC method and numerical solution of a continual model. Different cases were studied by varying the temperature of the stationary cylinder and the Knudsen number. The results of the continual model are obtained by setting the local value of Knudsen applied either to the boundary conditions. The comparison results showed that the slip boundary conditions with local Knudsen included improves the accuracy of the continual model.

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Numerical Solution of nonlinear Volterra Integral Equations of the Second Kind by Using Haar Wavelets

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Presenting a computational technique for nonlinear Volterra integral equations of the second kind is the main purpose of this paper. The method is based on Haar functions approximation. At the first step we present properties of Rationalized Haar functions, the operational matrix of integration, and then we produce operational matrix and Newton–Cotes nodes that reduce the computation of integral equations into some algebraic equations. These techniques computationally attractive and its applications are demonstrated through numerical examples.

Key words: Volterra integral equations, rationalized Haar function, operational matrix, product operation.

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Asymptotic Stabilization of a Chemostat Model of Plasmid-bearing, Plasmid-free Competition

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One of the modern developments of biotechnology in recent years is the laboratory production of substances by genetically modified organisms. The modification is obtained by inserting a plasmid, which codes for the production of the desired protein. These plasmid-bearing organisms are then growing in the chemostat. But during the reproductive process, with some probability, the plasmid might be lost, which introduces a second competitor, the plasmid-free organism [2]–[4].

The present paper continues the author's investigations in [1] on the following model of plasmid-bearing, plasmid-free competition in the chemostat

$$\begin{aligned} \dot{s} &= D(s^0 - s) - x_1\mu_1(s) - x_2\mu_2(s) \\ \dot{x}_1 &= x_1\left((1-q)\mu_1(s) - D\right) \\ \dot{x}_2 &= x_2(\mu_2(s) - D) + qx_1\mu_1(s), \end{aligned}$$

where $s(t)$, $x_1(t)$ and $x_2(t)$ are concentrations of the substrate, of the plasmid-bearing and plasmid-free organisms, s^0 is the input substrate concentration, D is the dilution rate in the chemostat, $\mu_i(s)$, $i = 1, 2$, are the specific growth rate (uptake) functions, and q is the plasmid loss probability ($0 < q < 1$). The main goal is to study the asymptotic stabilizability of the dynamic model under general assumptions on the uptake functions. Numerical simulations demonstrate the theoretical results.

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Exact Performance of (ρ, θ)-Hough Transform for Star Chain Images Processing

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The well known (ρ, θ)-interpretation [1] of Hough transform (HT) [2] is a projection technique that is most often treated in image processing considering its facilities to localize long stretched objects in a given image [3]. The definition of "*exact HT*" has been introduced for the both given grids $(x_{\mathrm{size}} \times y_{\mathrm{size}})$ and $(\rho_{\mathrm{size}} \times \theta_{\mathrm{size}})$ of the input image and of the HT result image respectively, considering the (ρ, θ)-HT like a Radon transform [4]. A few iterative approaches have been also proposed to approximate the exact HT through a balance among the *inner-noise-of-performance*'s level and the respective software effectiveness - programming duration and/or processing speed [4]. Unfortunately, these approaches become less and less acceptable with increasing of the input image grid as in the case of astronomical images, *e.g.*, when HT is applied for identification of flare objects in chain plate images [5]. In this work an analytic solution for the exact performance of (ρ, θ)-HT is proposed based on the results in [4]. Being not iterative the solution is enough simply programmable and is expected to be much more effective than the previous ones already mentioned.

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Numerical Investigation of Fluxon Interactions in Josephson Stacks

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The existence of stable coherent motions in different physical systems is a problem that attracts much attention in the last years. We study numerically the conditions under which a coherent (bunched) motion of fluxons allocated in different tunnel barriers of Josephson stacks is possible. The stability of such configuration is analyzed in detail. It is shown that a system of bunched fluxons reflects from the ends of the junctions without being destroyed. The influence of the number of the stacked junctions on such stable configurations is investigated. The investigation is made in the frame of the Sakai-Bodin-Pedersen model of N-stacked inductively coupled Josephson junctions. Finite difference methods are used to solve the system of nonlinear hyperbolic (perturbed sine-Gordon) equations. The accuracy and the stability of the method are investigated numerically.

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On The Existence Of Nonoscillatory and Oscillatory Solutions of Second Order Neutral Delay Impulsive Differential Equations with Constant Coefficients

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This paper is dealing with second order neutral delay impulsive differential equations with constant coefficients of the form

$$\begin{aligned} &[y(t) - cy(t-h_1)]'' + r_0 y(t) + r_1 y'(t) + p_0 y(t-h_1) \\ &\qquad + p_1 y'(t-h_1) = 0, \quad t \neq \tau_k,\ k \in N \\ &\Delta[y'(\tau_k) - cy'(\tau_k - h_1)] + s_1 y'(\tau_k) + u_1 y'(\tau_k - h_1) = 0, \quad k \in N \\ &\Delta[y(\tau_k) - cy(\tau_k - h_1)] + s_0 y(\tau_k) + u_0 y(\tau_k - h_1) = 0, \quad k \in N \end{aligned} \tag{1}$$

and

$$\begin{aligned} &[y(t) - cy(t-h_1)]'' + qy(t-h_2) = 0, \quad t \neq \tau_k,\ k \in N \\ &\Delta[y'(\tau_k) - cy'(\tau_k - h_1)] + Qy(t-h_2) = 0,\ k \in N \\ &\Delta[y(\tau_k) - cy(\tau_k - h_1)] + vy(\tau_k - h_2) = 0,\ k \in N. \end{aligned} \tag{2}$$

Characteristic equations of (1) and (2) are established and conditions, with respect to the arbitrary coefficients, about existence of nonoscillatory and oscillatory solutions of appropriate form of pulsatile exponent are obtained.

Key words: Neutral impulsive differential equation, oscillatory solution, nonoscillatory solution

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Spatio-temporal Modeling of Mosquito Dispersal. Applications in Vector Control

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Chikungunya is an uncommon disease and before the huge epidemic in Réunion Island in 2006, a French overseas department, our knowledges on this virus were small. The time and spatial evolution of a Chikungunya epidemic is strongly connected with the spreading of its principal vector in Réunion Island, Aedes albopictus.

In previous works, we have developed temporal models to assess the efficacy of different vector control tools, like adulticides, larvicides, mechanical control, which consists in reducing the breeding sites, and, even, the Sterile Insect Technique.

Now, we aim to take into account the spatial component. As a first step, we propose a partial differential equation to model female mosquito displacement, taking into account entomological knowledge: wind, attractors, like breeding sites and blood feeding sites, interactions between the landscape/vegetations and mosquito dispersal, ...

Then we split the females in two biological stages: one representing the females looking for breeding sites, and the other representing females looking for blood meals. This led to a system of two coupled partial differential equations. After some theoretical results, and using an appropriate approach, we provide numerical simulations and discuss the results.

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Frame Expansion in Banach Spaces and Its Applications for Stability

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As we know the theory of frames has applications such as signal processing, image processing, data compression, and *etc.* Frames are also used in wireless sensor networks, and geophones in geophysics measurements.

In this talk I discuss about frame expansion in Banach Spaces and show that the reconstruction formula: $f = \sum g_i(f) f_i$ in Banach frame, lets us to state and prove some useful stability of Banach frame under perturbations

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Motion of Particles in a Parabolic Flow near a Slip Wall

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The slip condition on modified surfaces at micro-scales may be exploited to facilitate the transport of suspensions of particles in micro-channels. Particles considered here are solid, spherical and suspended in a viscous fluid. The suspension is flowing along a plane wall and the fluid velocity profile is parabolic. The Reynolds number is low and Stokes equations apply. The boundary condition on particles is the classical no-slip condition. On the wall, a Navier slip condition applies [1].

Earlier related studies concern solutions of Stokes equations in bispherical coordinates for a sphere translating and rotating along a slip wall [2, 3] and suspended in a linear shear flow [3]. The problem of a fixed sphere in a quadratic flow is solved here. The case of a sphere moving in this flow field is superimposed to these earlier cases, by linearity of Stokes equations. Results are the force and torque on a fixed sphere and the velocities of translation and rotation of a freely moving sphere in a parabolic flow near a slip wall. The stresslet, that is the symmetric moment of surface stresses on the sphere, is also derived for a freely moving sphere in linear and parabolic shear flows. This quantity is useful in the calculation of the effective viscosity of a dilute suspension of freely moving spherical particles.

Finally, expansions of the various quantities for a large distance from the wall and for a low slip length are performed on the basis of the analytical solutions in bispherical coordinates, using the Mathematika© computer algebra software.

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Parameter Estimation for Gene Regulatory Network Modeling

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Gene expression is a complex biological process in which cells translate their genetic code into proteins. In this process, cells first transcribe their DNA into mRNA, and then the cell translates mRNA into proteins. Transcription factors regulate this process by increasing or decreasing the rate at which a cell transcribes a gene's DNA into RNA. In this presentation we discuss modeling the interactions between transcription factors thought to play a role in controlling the environmental stress response of cold shock in Saccharomyces cerevisiae. The dynamical system we develop is compared to published yeast cold shock microarray data for the purposes of parameter estimation and sensitivity analysis.

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Atmospheric Composition of the Balkan Region and Bulgaria – Some Numerical Experiments

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The present work aims at studying the local to regional atmospheric pollution transport and transformation processes over the Balkan Peninsula and at tracking and characterizing the main pathways and processes that lead to atmospheric composition formation in the region. The US EPA Models-3 system is chosen as a modeling tool. As the NCEP Global Analysis Data with 1 degree resolution is used as meteorological background, the MM5 and CMAQ nesting capabilities are applied for downscaling the simulations to a 9 km resolution over Balkans and 3 km over Bulgaria. The TNO emission inventory is used as emission input. Special pre-processing procedures are created for introducing temporal profiles and speciation of the emissions.

The air pollution pattern is formed as a result of interaction of different processes. Therefore the Models-3 "Integrated Process Rate Analysis" option is applied to discriminate the role of different dynamic and chemical processes for the pollution formation. The processes that are considered are: advection, diffusion, mass adjustment, emissions, dry deposition, chemistry, aerosol processes and cloud processes/aqueous chemistry.

Some results from several emission scenarios which make it possible to evaluate the contribution of different SNAP categories are demonstrated as well. Numerical experiments for quantifying the role of biogenic emissions are also carried out.

Key words: Air pollution modeling, US EPA models-3 system, multi-scale modeling, process analysis, emission scenarios

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High Fidelity Finite Length Markov Chain Walks

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Given an irreducible Markov chain model, what is the complete set of the highest fidelity state sequences of a given length (finite length walks), that the model is able to produce? Here, highest fidelity property means : 1) all sequences in the set have the same relative transition frequency matrix ; 2) according to some measure, the relative transition occurrence frequency matrix is as close to the model stochastic matrix as possible ; 3) each sequence starts with a state that has nonzero initial probability. A three stage approach for generation of such complete set is discussed : 1) calculation of the exact absolute transition occurrence numbers matrix, which in general, consists of non-integer values ; 2) application of controlled matrix rounding to the latter in order to obtain integer values for all transition occurrence numbers, which introduces a bias leading to the fidelity issue ; 3) generation of all sequences that satisfy the constraints for state transition occurrence numbers from the integer matrix, which can be done by any constraint preserving permutation group generating technique. This paper is focused on the first stage an original linear algebraic solution to the problem is proposed. Three distinct and mutually complementary intuitive constraint aspects of the finite length Markov chain walks are formulated and formalized as three independent systems of linear equations with the exact absolute state transition occurrence numbers being the unknowns. Each system suffers from linear dependency among its equations. After proper elimination of certain part of each system and union of what is left of the three, one aggregated system is constructed. The conditions for the existence of such system's solution are investigated and its uniqueness is proven. The influence of solution's values over the existence of respective walks is considered. Several interesting properties of the system and its solution are discussed, some of them with proofs.

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Two Level Method for Discontinuous Galerkin Discretizations of Elliptic Problems with Highly Varying Coefficients

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In this talk we present a generalized hierarchical bases for symmetric positive definite matrices arising from discontinuous Galerkin discretization of second-order partial differential equations (PDE) with highly varying coefficients. In the well-established theory of hierarchical basis multilevel methods one basic assumption is that the PDE coefficients are smooth functions on the elements of the coarsest mesh partition. However, as it is shown (for the two-level basis of the scalar elliptic problem), the generalized hierarchical basis yields a robust splitting with respect to jump discontinuities of the PDE coefficient at arbitrary element interfaces on the finest mesh.

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On Soliton Equations and Soliton Interactions

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We will consider two important classes of soliton equations: the N-wave equations

$$i\left[J, \frac{\partial Q}{\partial t}\right] - i\left[I, \frac{\partial Q}{\partial x}\right] - [[J, Q], [I, Q(x,t)]] = 0, \tag{1}$$

and the multicomponent nonlinear Schrödinger (MNLS) equations related to the **BD.I** type of symmetric spaces

$$i\vec{q}_t + \vec{q}_{xx} + 2(\vec{q}^{\,\dagger}, \vec{q})\vec{q} - (\vec{q}, s_0\vec{q})s_0\vec{q}^{\,*} = 0. \tag{2}$$

Here for $2r-1$-component vector function $\vec{q}(x,t)$ the constant matrix

$$s_0 = \sum_{j=1}^{2r-1} (-1)^{k+1} E_{k,2r-k}$$

and $(E_{mn})_{jl} = \delta_{mj}\delta nl$.

The N-wave equations describe nonlinear $\chi^{(2)}$-optical media. The MNLS equation (2) for $r=2$ describe, in particular, a Bose-Einstein condensate with spin $F=1$.

These two classes of soliton equations allow different types of soliton solutions. We will outline them and will analyze their interactions.

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Moments and Micromorphic Continua

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Among the many contributions of the late A. C. Eringen to continuum mechanics is his extensive collection of work on *micromorphic* continua. We recall that, as a generalization of the celebrated Cosserat continua, each point in such a material manifold is endowed with a local microstructure whose kinematics are generally distinct from the motion of the underlying manifold itself. The generalized forces or stresses that expend kinematic power are elements of the dual space to the tangent space or space of generalized velocities associated with the micromorphic kinematics, and they generally appear as higher-order moments of ordinary force or (Cauchy) stress.

As one one approach to conceptualizing the physical origins of such a medium, one may consider it to arise from an ordinary ("simple") continuum fragmented into microdomains endowed with the standard smooth kinematics. This is the approach apparently favored by Eringen, who obtained micromorphic kinematics and stress from various integral moments over microdomains.

In a recently published paper, this author [4] gives a comparison of Eringen's [1] moment balances for micromorphic continua with P. Germain's [2] generalized momentum balances based on virtual work principles and also with those derived by a two-scale Fourier analysis of heterogeneous media. It has not been possible to establish a clear-cut correspondence between any of the three methods, and some tentative explanations are given.

As another approach to the physics of micromorphic continua, one may attempt to proceed from the Newtonian mechanics of point particles and to obtain Cauchy stress and higher moment stresses from moments of Newton's law. To this end, a construction of path-moments of density fields serves to establish a source-flux duality in continuum balances, which *inter alia* establishes a fairly direct connection between Newton's and Cauchy's laws and provides an expression for stress suggested by the statistical mechanics of point-particles [5], [3]. Furthermore, the present approach appears to provide a general mathematical foundation for the "peridynamics" of Stilling and coworkers [6]. It remains to be seen whether this approach can be easily adapted to the derivation of moments associated with micromorhpic continua.

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Framework for Service Composition in g-Lite

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g-Lite is a Grid middleware, currently the main middleware installed on all clusters in Bulgaria. The scientists use the middleware in order to solve problems, which require a large amount of storage and computational resources. On the other hand, they work with complex processes, where job execution in Grid is just a step of the process. That is why, it is strategically important g-Lite to provide mechanism for service compositions and business process management. Such mechanism is not specified yet. In this article we propose a framework for service composition in g-Lite. We discuss business process modeling, deployment and execution in this Grid environment. The examples used to demonstrate the concept are based on some IBM products.

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The Static and Dynamic Stability of Carbon Nanotubes

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This lecture reviews recent research studies on the stability of carbon nanotubes. We first introduce the audience to the structure and properties of carbon nanotubes. Then, various instabilities exhibited by carbon nanotubes are presented. In the second part of this lecture we introduce the two most used methods for stability analysis of carbon nanotubes, *i.e.*, continuum models and atomistic simulations. We close the talk with delineating the main factors, such as dimensions, temperature, strain rate, and boundary conditions, influencing the stability of carbon nanotubes. It is hoped that this work provides current knowledge on the stability of carbon nanotubes, reviews the analytical as well as computational methods for determining the margins of stability, and inspires researchers to further investigate the instabilities of carbon nanotubes for better design and practical applications.

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How to Use HPC Resources Efficiently by a Message Oriented Framework

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The Bulgarian HPC infrastructure consists of several small HPC clusters with Intel CPUs and Infiniband interconnection (total more than 14000 logical cores) grouped around the powerful supercomputer – BlueGene/P with 8192 CPU cores. In addition some servers equipped with powerful GPU are available for applications that can take advantage of them. The coordinated use of such resources by one application faces significant challenges due to the heterogeneity of the resources and the networking and security constraints.

In order to facilitate the coordinated use of all these resources where each resource is used for the parts of the application where it is most efficient, we have developed a framework that allows the researchers to interconnect resources of the above types with minimal overhead. In this paper we describe the architecture of the system and demonstrate its effectiveness for a semiconductor modeling application, showing numerical and timing results.

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Modeling the Effects of Drug Binding on the Dynamic Instability of Microtubules

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We propose a stochastic model that accounts for the growth, catastrophe and rescue processes of steady state microtubules assembled from MAP-free tubulin in the possible presence of a microtubule associated drug. As an example for the latter, we both experimentally and theoretically study the perturbation of microtubule dynamic instability by S-methyl-D-DM1, a synthetic derivative of the microtubule-targeted agent maytansine and a potential anticancer agent. We find that among drugs that act locally at the microtubule tip, primary inhibition of the loss of GDP tubulin results in stronger damping of microtubule dynamics than inhibition of GTP tubulin addition. On the other hand, drugs whose action occurs in the interior of the microtubule need to be present in much higher concentrations to have visible effects.

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A Non-standard Finite Element Method based on Boundary Integral Operators

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We present and analyze a new non-standard finite element method based on element-local boundary integral operators that permits polyhedral element shapes as well as meshes with hanging nodes. The method employs elementwise PDE-harmonic trial functions and can thus be interpreted as a local Trefftz method. The construction principle requires the explicit knowledge of the fundamental solution of the partial differential operator, but only locally, *i.e.*, in every polyhedral element. This allows us to solve PDEs with elementwise constant coefficients.

Using the diffusion equation as a model problem, we provide a rigorous H1- and L2-error analysis of the method for smooth and non-smooth solutions under quite general assumptions on the geometric properties of the polyhedral elements. Numerical results confirm our theoretical estimates and demonstrate the applicability of the scheme to more complicated problems like convection-diffusion equations.

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Numerical Solvers for Maximal Solution to a Set of Discrete-time Algebraic Riccati Equations

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Consider the set of coupled algebraic discrete time Riccati equations of the form:

$$x_i = \sum_{k=0}^{r} A_i^{\mathrm{T}}(k)\Xi_i(X)A_i(k) + Q_i - \left(\sum_{k=0}^{r} A_i^{\mathrm{T}}(k)\Xi_i(X)B_i(k) + L_i\right) \times \left(R_i + \sum_{k=0}^{r} B_i^{\mathrm{T}}(k)\Xi_i(X)B_i(k)\right)^{-1} \left(\sum_{k=0}^{r} B_i^{\mathrm{T}}(k)\Xi_i(X)A_i(k) + L_i^{\mathrm{T}}\right) \quad (3)$$

where $X = (X_1, X_2, ..., X_N)$ and $\Xi_i(X) + \sum_{j=1}^{N} p_{ij}X_j,\ i = 1, ..., N$. The considered system of matrix equations arise in control theory as an important tool for solving many optimization control and filtering problems as an example it is the linear-quadratic problems. In Dragan *et al* [1] a new procedure for numerical computation of the maximal solution and the stabilizing solution is proposed. In addition, we apply the known decoupled Stein iteration [2] for solving (1). Moreover, an LMI optimization problem is considered as a very important tool for solving (1).

In this talk we investigate the numerical effectiveness of the above mentioned methods and we compare their numerically. For this purpose numerical simulations are executed.

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Postural Stability Analysis and PID Controller Synthesis for Movement Coordination in a Multi-segment Biomechanical Model

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Biomechanics of the human body are characterized by an abundance of degrees of freedom that must be controlled and managed to realize a coordinated movement. Quiet standing involves incessant postural adjustments aimed to counter the multidimensional disturbances to standing posture. Motor control of postural and voluntary movements is executed as a series of motor programs that specify muscle synergy, sequencing of contraction, relative timings, and durations. The sensorimotor cortex translates these programs into mechanical stiffness at the joints, movement direction, velocity and end points. The spinal cord helps set the muscle tone and stiffness, and the sensitivity of primary sensory organs, the muscle spindle (MS) and the golgi tendon organ (GTO).

Postural stability constitutes an important attribute of the musculoskeletal-proprioceptive apparatus. Postural stability is enabled by the biological servomechanism involving muscles, stretch receptors and neural pathways. The servo system generates reflexes aimed at relieving muscle tension, and returning muscle tone to its preset level. Passive viscoelasticity of the musculo-tendon complexes (MTCs) is necessary but not sufficient for stability. Stability augmentation is delivered through persistent central nervous system (CNS) involvement via spinal reflexes, internal model control, and anticipatory postural adjustments (APA). Active control of muscle stiffness and tone relies on proprioceptive feedback to dynamically modulate muscle afferents and maintain balance. However, intrinsic delays in the reflex pathways and the low-pass characteristics of the muscle response tend to limit the effectiveness of active mechanisms of postural stabilization. How then standing humans are able to maintain balance remains an open and intriguing question. The use of mathematical models to provide insight into neurophysiology has a long history. Control of balance in human upright standing is particularly well suited for modeling, and is also a popular experimental paradigm. In this study we consider a simplified characterization of the postural control system that embraces two broad components: one representing the musculoskeletal dynamics in the sagittal plane, and the other representing proprioceptive feedback and the decision making modalities of the CNS. Specifically, a four-segment sagittal model of the physiological system is developed that includes important physiological constructs such as Hill-type muscle model, active and passive muscle stiffness, force feedback from the GTO, muscle length and rate feedback from the MS, and transmission latencies in the neural pathways. A proportional-integral-derivative (PID) controller for each individual degree of freedom (DOF) is assumed to represent the CNS analogue in the modeling paradigm. The overall control structure for postural stability and movement coordination consists of three components: 1) a reference trajectory; 2) a set of autonomous PID controllers; and, 3) the position, velocity, and force feedback loops with physiological latencies.

We present analytical and simulation results to show that the proposed representation adequately shapes a postural control that: a) possesses good disturbance rejection and trajectory tracking, b) is robust against feedback latencies and torque perturbations, and c) is flexible to embrace changes in the musculoskeletal parameters. We additionally present detailed sensitivity analysis to show that control under conditions of limited or no proprioceptive feedback results in: a) significant reduction in the stability margins, b) substantial decrease in the available stabilizing parameter set, and c) oscillatory movement trajectories. Overall, these results suggest that anatomical arrangement, active muscle stiffness, force feedback, and physiological latencies play a major role in shaping motor control processes in humans.

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Nonlinear Acoustic Propagation in Thermoviscous, Thermally Relaxing Fluids: Shock Formation and Viscosity Mitigation

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We consider the propagation of finite-amplitude acoustic waves in thermoviscous fluids that also exhibit thermal relaxation, meaning that they also support the propagation of second-sound (i.e., thermal waves). Under the assumption that the thermal flux vector satisfies the Maxwell-Cattaneo-Christov law, a well known generalization of Fourier's law that includes the effects of thermal inertia, we derive the corresponding weakly nonlinear equation of motion, which is a generalization of the classic Kuznetsov equation, in terms of the velocity potential. By analyzing the traveling wave solution of this equation, it is shown that a critical value of ?0, the thermal relaxation time, occurs and that this critical value is an upper bound on ?0. It is also shown that as ?0 approaches its critical value (from below), the effects of viscous and thermal dissipation begin to vanish. We then use singular surface theory to determine how an input signal in the form of a shock wave evolves over time, and for different values of the Mach number. Numerical methods are then used to illustrate the analytical findings. In particular, it is shown that the shock amplitude exhibits a transcritical bifurcation; that a stable, nonzero equilibrium solution is possible; and that a Taylor shock (or kink), in the form of a "tanh" profile, can emerge from the input shock signal. Finally, an application related to the kinematic-wave theory of traffic flow is also discussed. (Work supported by ONR/NRL funding.)

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Immersed FEM for Elliptic Interface Problems with Non-homogeneous Jump Condition of Special Type

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In this paper an elliptic interface problem with continuous solution and jump of the flux, that depend on the solution at the interface is considered. A numerical method, based on the Immersed Interface Finite Element Method is developed. The special jump condition in the mathematical model leads to using of a modified basis functions near the interface. Quadratic immersed finite element space is developed. Theoretical analysis are presented. Several test examples demonstrate the accuracy of the method for the linear and nonlinear case of local source and discontinuous coefficients. The results confirm third order of the numerical solution in maximum norm and second order in Sobolev norm.

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Monte Carlo Simulation of Electron Transport using a Class of Sequences Generating Permutations

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Sequences in finite fields whose terms depend in a simple manner on their predecessors are of great importance for a variety of applications. Of particular interest is the case where the terms depend linearly on a fixed number of predecessors, resulting in a linear recurring sequence.

In this paper we present a class of sequences generating permutations [1] and study their applicability for Monte Carlo simulation of electron transport. These sequences are developed for other applications but our analysis and experiments show that they outperform some of the PRN generators for our problem.

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On the Mathematical Model of Cancer Growth in Connection to the New Capillary Network Formation at the Human Brain

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The mathematical model of tumor growth is constructed taking into the account the new capillary vessel formation and competition of normal and tumor cells for the nutrients supply. The tumor growth in a human body imply oxidative stress and as a result new capillary vessels formation from the bones morrow progenitor cells. We choose the definite volume of the brain, where tumor originated. The system of differential equations is obtained. At this system nutrients consumption rates of normal and tumor cells, number of the capillaries, necrotic factors and immune reaction of the body are taken into the account. The system is investigated numerically. Computer simulations are given. The designated project has been fulfilled by financial support of the Georgia Rustaveli Foundation (Grant #GNSF/ST08/3-395). Any idea in this publication is possessed by the author and may not represent the opinion of the Foundation itself.

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Paradoxes of Dissipation-Induced Instabilities in Mechanics and Physics

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In 1952 Ziegler found that an infinitesimally small amount of damping leads to a finite change in the stability domain of a two-link pendulum loaded by the follower force. In 1956 Bottema resolved this destabilization paradox by means of the Whitney umbrella singularity that as he established exists on the stability boundary of the damped Ziegler's pendulum. I will talk about extensions of this result to general finite dimensional and continuous circulatory systems as well as to the gyroscopic systems with small damping and non-conservative positional forces.

Examples of similar paradox phenomena from rotor dynamics, continuum mechanics and magnetohydrodynamics will be considered in detail. A broad overview of the achievements in this field over the last half a century will be given.

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Comparison of Three Methods to Estimate Wind Speed Distribution

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This paper compares three methods of recovering a probability density function (pdf) - maximum likelihood method (ML), maximum entropy method (ME) and fractional calculus method (FC) that uses complex moments to represent the pdf. The comparison is based on an analysis of wind data measurements recorded in North Bulgaria for the project DO02-48/10.12.2008, NSF, Ministry of Education. The model selection criteria used in this study are root mean square error (RMSE) and cross entropy (CE). The results show that the second two methods fit better to wind speed frequency distribution

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Iterative Methods for Solving Nonlinear Parabolic Problem in Pension Saving Management

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In this work we consider a nonlinear parabolic equation, obtained from Riccati like transformation of the Hamilton-Jacobi-Bellman equation, arising in pension saving management. We discuss two numerical iterative methods for solving the model problem – fully implicit Picard-Newton method and implicit-explicit quasi-Newton method, which preserves the parabolic characteristics of the differential problem. Numerical experiments for comparison the accuracy and effectiveness of the algorithms are discussed. Finally, observations are given.

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Mathematical Modeling of Phagocyte Antibacterial Activity on Medical Implant Surfaces

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Device-centered infection is a devastating complication associated with surgical implants. Many implants have to be removed because of infection, thrombosis, and excessive inflammation, among others. In this talk, we discuss discrete and continuous mathematical models that represent the complex interaction between S. epidermidis and Neutrophils on medical implants pre-coated with mixtures of different proteins and/or antibiotics. The main modeling assumption is that species and physical state of adsorbed host proteins control the host reactions to the implants. An approach based on transition probabilities is used for upscaling the discrete cellular automata models to the continuous PDE models. The proposed new models are validated through a series of comparisons of numerical simulation results with real biomedical data. The knowledge gained from this work can be used to establish criteria and conditions required for fabrication of antimicrobial implants that improve the body's defense against bacterial infections while simultaneously decreasing the adverse foreign body reactions.

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Reliable FEM Approximations for a Model of Cell-matrix Interactions and Cell Migration

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The model equations describe interactions between tumor cells and microenvironmental factors such as the extracellular matrix (ECM) and matrix degradative enzymes (MDEs). The interacting variables are n - tumor cell density, f - ECM density, and m - MDEs concentration. The unknown functions $n = n(x,t)$, $f = f(x,t)$ $m = m(x,t)$ satisfy a quasilinear parabolic system.

An additional biological condition to the obligatory requirement of convergence of computed approximations to the exact solution n, f, m is the requirement the approximations to be nonnegative (a continuous maximum principle).

In this paper we derive and discuss sufficient conditions for providing validity of the discrete maximum principle. The theoretical analysis is supported by numerical experiments.

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A New Conservative Finite Difference Scheme for Boussinesq Paradigm Equation

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We consider the Boussinesq Paradigm Equation

$$\frac{\partial^2 u}{\partial t^2} = \Delta u + \beta_1 \Delta \frac{\partial^2 u}{\partial t^2} - \beta_2 \Delta^2 u + \alpha \Delta f(u), \quad x \in \mathbb{R}^2,\ t > 0,$$
$$u(x,0) = u_0(x), \quad \frac{\partial u}{\partial t}(x,0) = u_1(x),$$

where α, β_1, β_2 are positive constants and the solution u satisfies the asymptotic boundary conditions $u(x,t) \to 0$, $\Delta u(x,t) \to 0$ as $|x| \to \infty$. Typically, the nonlinear term is $f(u) = u^2$.

We obtain the numerical solution of this problem applying a new family of conservative finite difference schemes.

We modify the approximation of the nonlinear term in order to get second order of convergence in space and in time. The modification is not trivial due to the conservativity of the scheme.

Our numerical experiments show clear advantage of the new scheme over the known conservative scheme in precision.

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Theory of Quantum Transitions Based on Nonadiabatic Dressed States

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A theory of quantum transitions based on nonadiabatic dressed states (NADSs) is presented. The derivation of the NADSs of a quantum system interacting with an external electromagnetic field and the environment is outlined. The electromagnetic field is presented in an explicit form employing the carrier-envelop concept. The interaction with the environment is described by means of a dumping term. The amplitude and phase of the field are arbitrary functions of time, subject only to a generalized adiabatic condition. The field time derivatives and the dumping play role of non-adiabatic factors, which, according to the adiabatic theorem of quantum mechanics, are responsible for the quantum transitions. The main matrix elements within the NADSs are found. The transition probabilities of the radiative and the nonradiative quantum transitions are found in terms of closed form expressions.

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A Model of Within-cell Virus Replication

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Viruses are the simplest living organisms. In order to survive, a virus has to successfully invade a host cell, overcome cellular degradation mechanisms, produce progeny and export it to infect other cells; eventually evade immune response and jump to a new host to start the cycle again. The first challenge to virus survival is successful reproduction in the host cell. For RNA viruses, such reproduction includes a balance between several competing processes: production of RNA-derived RNA polymerase (RdRp), production of viral protein, RNA replication by the RdRp, formation of virions by combination of genomic RNA and structural viral protein and degradation of these products. Here we design a model representing these processes for positive-sense single stranded viruses (such as the family of Picorna or Flavi viruses) as a system of ODEs derived from stoichiometric enzyme-substrate reactions and explore the asymptotic dynamics of the model. The possible regimes are (1) virus extinction, (2) stable steady state and (3) a regime where RNA and RdRp are produced in excess (tend to infinity in the model) while the structural protein is fully utilized (converges to 0). If the net production of virions is a measure of virus fitness (such a claim is supported by the view that larger virus populations can maintain higher diversity and therefore be more adaptable), then we show that viruses that have evolved to utilize scenario (3) have higher fitness than viruses that establish equilibrium progeny production within the cell.

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Comparison of Two Techniques for Radio-Frequency Hepatic Tumor Ablation through Numerical Simulation

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We simulate the thermal and electrical processes, involved in the radio-frequency (RF) ablation procedure. RF ablation is a low invasive technique for the treatment of hepatic tumors, utilizing AC current to destroy the tumor cells by heating. The destruction of the cells occurs at temperatures of $45 - 50°$ C. The procedure is relatively safe, as it does not require open surgery. The surgeon places a RF probe inside the tumor and initiates RF current on it.

In this study, we take into account the observed fact, that the electrical conductivity of the hepatic tissue varies during the procedure. With the increase of the temperature, a gradual increase of the conductivity is observed, followed by a drastic drop. This variation was neglected in some previous studies.

The mathematical model consists of two parts – electrical and thermal. The energy from the applied AC voltage is determined first, by solving the Laplace equation to find the potential distribution. After that, the electric field intensity and the current density are directly calculated. Finally, the heat transfer equation is solved to determine the temperature distribution. Heat loss due to blood perfusion is also accounted for.

The first considered technique uses a probe with retractable electrodes, deployed in an umbrella-like shape. The second technique uses a needle as a probe and saline solution is constantly injected during the procedure. This is done in order to maintain a more stable electrical conductuvity around the probe. We compare the size and shape of the region where cells are destroyed during ablation procedures using the above techniques.

The simulations were performed on the IBM Blue Gene/P massively parallel computer.

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On the Use of Numerical Data for Evaluating the Parameters of the a-priori Estimates for the Onset of Blow up in Boussinesq Paradigm Equation

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While it is well known that the blow up of the weak solutions to Boussinesq Paradigm Equation depends not only on the initial energy of the data but also on their profile, the constants that appear in the *a-priori* estimates cannot be found from the theoretical analysis.

In this paper, we find the way to employ the information from direct numerical calculations to find tighter estimates for these constants both from below and from above. In this way we can find the intervals for the parameters in which exist globally in time with arbitrary initial data blows up.

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Resonant Behavior of a Fractional Oscillator with Random Damping Micro-channel

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As the fractional oscillator (FO) is the simplest toy model for different phenomena in viscoelastic media it is a typical theoretician's paradigm for various fundamental ideas in the fields of science and engineering. We investigate the behavior of a FO in the case of a fluctuating damping coefficient. Although the behavior of the FO with a random eigenfrequency has been investigated in some detail [1], it seems that analysis of the potential consequences of interplay between a random damping and memory effects incorporated in the FO is absent in literature. This is quite unjustified in view of the fact that the importance of multiplicative fluctuations of damping for various natural systems, *e. g.*, for water waves influenced by a turbulent wind field, has been well recognized [2]. Thus motivated, we consider a FO with a power-law memory kernel subjected to an external periodic force. The influence of the fluctuating environment is modeled by a multiplicative dichotomous noise (fluctuating damping) and an additive noise. The main purpose of this work is to provide exact formulas for the analytic treatment of the dependence of spectral amplification on the system parameters: viz. the noise correlation time, noise amplitude, memory exponent, and driving frequency. Based on those exact expressions we demonstrate that stochastic resonance is manifested in the dependence of the spectral amplification upon the noise parameters. Furthermore, we will show that in certain parameter regions the FO exhibits a multiresonance behavior versus the driving frequency. Moreover, a couple of critical memory exponents are found which mark the transitions between various dynamical regimes of the oscillator.

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Highly Parallel Alternating Directions Algorithm for Time Dependent Problems

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We consider the time dependent Stokes equation on a finite time interval and on a uniform rectangular mesh, written in terms of velocity and pressure.

A parallel algorithm based on a new direction splitting approach is developed. Here, the pressure equation is derived from a perturbed form of the continuity equation in which the incompressibility constraint is penalized in a negative norm induced by direction splitting. The scheme used in the algorithm is composed by two parts: velocity prediction and pressure correction. This is a Crank-Nicolson-like two-stage time integration scheme for two and three dimensional parabolic problems in which the second-order derivative with respect to each space variable is treated implicitly while the other is made explicit at each time step. In order to achieve a good parallel performance the solution of the Poison problem for the pressure correction is replaced by solving a sequence of one-dimensional second order elliptic boundary value problems in each spatial direction.

The parallel code is developed using the standard MPI functions and tested on modern parallel computer systems. The performed numerical tests demonstrate the level of parallel efficiency and scalability of the direction-splitting based algorithm.

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Stability Analysis and Dynamically Consistent Numerical Schemes for the SIS Model and Related Reaction Diffusion and Volterra Integral Equations

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The classical SIS epidemiological model is extended in three directions: (a) The number of adequate contacts per infective in unit time is assumed to be a function of the total population in such a way that this number grows less rapidly as the total population increases; (b) A diffusion term is added to the SIS model and this leads to a reaction diffusion equation, which governs the spatial spread of the disease; (c) The SIS is formulated as a Volterra integral equation. With the parameter R_0 representing the basic reproduction number, it is shown that $R_0 = 1$ is a forward bifurcation for the model (a), with the disease-free equilibrium being globally asymptotic stable when R_0 is less than 1, while the model (c) can exhibit the backward bifurcation phenomenon. In the case when R_0 is greater than 1, traveling wave solutions are found for the model (b). Nonstandard finite difference (NSFD) schemes that replicate the dynamics of the continuous models are presented. In particular, for the model (a), nonstandard versions of the Runge-Kutta methods having high orders of convergence are investigated.

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The New Approach to Analysis of Chaotic Dynamics in Hamiltonian and Conservative Systems

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The new approach is offered for the analysis of chaotic dynamics of solutions of complex perturbed conservative (in particular, Hamiltonian) systems. This approach implies the construction of an approximating extended two-parametrical dissipative system of the equations, whose stable solutions (attractors) are as much as exact approximations to solutions of the original conservative system. On the basis of the carried out numerical calculations for several three- and four-dimensional conservative systems and Hamiltonian systems with two and three degrees of freedom, it is shown, that in all these systems transition to chaos occurs not through destruction of two-dimensional or three-dimensional tori of unperturbed system, but, conversely, through a generation of complex two-dimensional tori around of cycles of extended dissipative system and through the infinite cascades of bifurcations of generation of new cycles, tori and singular trajectories according to the Feigenbaum-Sharkovskii-Magnitskii (FSM) theory.

Keywords: Hamiltonian and conservative systems, chaos, separatrix manifold, theory FSM

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Steganography, Error Control Codes and the Relation with HPC

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A method for embedding messages in images is described. It is based on the erasure error-control codes. The use of grid and/or multiprocessor systems during the process of choosing parameters of the embedding is discussed.

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On Semi-coarsening Multilevel Preconditioning of Higher Order FEM Systems

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While a huge amount of papers are dealing with robust multilevel methods and algorithms for linear FEM elliptic systems, the related higher order FEM problems are much less studied. It is well known that the standard hierarchical basis two-level splittings deteriorate for strongly anisotropic quadratic FEM problems. First robust multilevel preconditioners for biquadratic anisotropic FEM elliptic problems were recently developed. We study the behavior of the constant in the strengthened CBS inequality for semi-coarsening mesh refinement which is a quality measure for hierarchical two-level splittings of the considered biquadratic FEM stiffness matrices. Some new results for the case of balanced semi-coarsening is of a particular interest. The presented theoretical estimates are supported by numerically computed CBS constants for a rich set of parameters (coarsening factor and anisotropy ratio). Combining the proven uniform estimates with the theory of the Algebraic MultiLevel Iteration (AMLI) methods we obtain optimal order multilevel algorithms whose total computational cost is proportional to the size of the discrete problem with a proportionality constant independent of the anisotropy ratio. The provided comparative analysis of the pure and balanced semi-coarsening algorithms addresses both computational complexity and parallel implementation issues.

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Statistical Simulation of Gas Flows through Short Rough Microchannels

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Over the last decade a great impulse has been given to the development of micromechanical systems. Many kinds of micro-sensors, micro-actuators, micro heat exchanges have been developed and have been placed in the market. Understanding of fluid and gas flow behavior inside such systems is crucial for further development and the numerical modeling plays an important role. Due to a very small size of the micro-system rarefaction effects have to be taken into account, which often requires an application of both continuum and kinetic numerical approaches. For micro-flow applications the surface roughness affects flow properties because its height can be up to 10characteristic scale. Deterministic and statistical methods have been developed to take the roughness into account. The first methods model a gas flow using a realistic surface shape when the latter treat the surface using a statistical description. One of the example of the statistical methods is an application of porous media model for rough microchannels done by the coauthor.

The purposes of the paper are to compare both methods by computing gas flows through short rough microchannels and compare results with available numerical data, for example, Navier-Stokes results obtained for the roughness as a series of triangular obstructions. The well-known software, SMILE, has been used for computations. It is based on kinetic approach and it allows one to perform CPU resource consuming computations for microchannels with an arbitrary shape. The short microchannels require the special inlet boundary conditions to consider the entrance effects, correctly. A porous media model has been implemented in the software as a statistical method.

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On the Mathematical Modeling of Microbial Growth

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We propose a new approach to mathematical modeling of microbial growth. Our approach differs from familiar Jacob-Monod type models by considering two phases in the physiological states of the microorganisms. Such an approach may be useful in the modeling and control of biotechnological processes, where microorganisms are used for various biodegradation purposes and are often put under extremely unfavorable conditions, such as prolonged depletion or excess of nutrients. Some numerical experiments are performed in support to our modeling approach.

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Seasonality in Avian Influenca H5N1

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Avian influenza H5N1 has been infecting poultry and humans in many countries since 2003.

The cases follow a seasonal pattern with peaks in the winter months, December through March. This pattern is puzzling as most of the human cases occur in equatorial countries where even the seasonal human flu occurs with different pattern. We hypothesize three different mechanisms that may be responsible for the seasonality in H5N1 cases: (1) seasonality in direct transmission in domestic birds; (2) seasonality introduced by migratory patterns in wild birds; (3) seasonality introduced by environmental transmission of H5N1. We incorporate all these types of seasonality one by one or in combination. We fit the resulting models to the cumulative number of human cases reported by the World Health Organization for the period January 2005-December 2009. We compare the models based on their least square error.

Furthermore, we use the best fitted models to project the cumulative number of human cases of H5N1 through 2010 and compare it with the incoming data. The best fitted models show good agreement with future data. This is a preliminary report.

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Third Order Convergence for Nonlinear Impulsive Differential Equations with a Nonlinear Three-point Boundary Condition

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The method of generalized quasilinearization has been extended for nonlinear impulsive differential equations with a nonlinear three-point boundary condition. Natural lower and upper solutions under suitable conditions are considered. We have obtained two sequences which converge uniformly and monotonically to the unique solution of the nonlinear impulsive differential equations. We further prove that the rate of convergence is cubic.

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Traveling Wave Solutions of the Gardner Equation

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The Gardner equation, which is sometimes called also a combined KdV–mKdV equation, is known in the mathematical literature since the late sixties of 20-th century. Initially, it appeared in the context of the construction of local conservation laws admitted by the KdV equation. Later on, the Gardner equation was found to be applicable in various branches of physics (solid-state and plasma physics, fluid dynamics and quantum field theory). Here, we examine the traveling wave solutions of the Gardner equation and derive the full set of solutions to the corresponding reduced equation in terms of the Weierstrass and the Jacobi elliptic functions. A number of possible applications of the obtained results are pointed out.

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Rotations and Some Mechanical Applications

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The present work presents our research activity concerning the problems of modelling and control of a rigid-body motion and its applications in multibody mechanical systems. Since the treatment of the above topic is quite sensitive with respect to the different parameterizations of the rotation group in three dimensional space $SO(3)$ and since the properties of the parameterization more or less influence on the efficiency of the dynamic model, a full analysis of the different ways of $SO(3)$ group representation is suggested.

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Sensitivity Study of the Ozone Concentrations Output from the Unified Danish Eulerian Model in Parallel and Grid Environment

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Variance-based techniques are used to compute Sobol' global sensitivity indices in sensitivity study of the output ozone concentrations, calculated by the Unified Danish Eulerian Model. This method requires a lot of output data from computationally expensive numerical experiments with a specially adapted for the purpose version of UNI–DEM. It has been successfully implemented and run on the most powerful parallel supercomputer in Bulgaria - IBM BlueGene/P, as well as an user implementation in the Grid environment. The results have been used to construct mesh-functions according to variations of an extended set of chemical reactions' rate coefficients. The mesh-functions are to be used further in sensitivity analysis of the model by using Monte Carlo algorithms.

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A Beam-Fourier Technique for the Numerical Investigation of 2D Nonlinear Thermoconvective Flows

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In the current work, we develop a numerical method suitable for treating the problem of nonlinear two-dimensional flows in rectangular domains. For the spatial approximation we employ the Fourier- Galerkin approach. More specifically, our basis functions are products of trigonometric and Beam functions. This choice means that the solutions automatically satisfy the boundary and periodic conditions in the x and y directions respectively.

The accuracy of the method is assessed by applying it to a model problem which admits an exact analytical solution. The numerical and analytic solutions are found to be in good agreement. The method is then applied to the problem of two-dimensional thermoconvective flow in a vertical slot subject to both horizontal and vertical temperature gradients. The convergence rate of the spectral coefficient is found to be fifth-order algebraic in the x-direction and geometric in the y-direction, confirming the efficiency and speed of our technique.

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Spectral Formulation for the Solution of Full-wave Scattering from a Conducting Wedge Tipped with a Corrugated Cylinder

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A spectral mode-matching technique is formulated to solve for the full-wave scattering of a corrugated cylinder-tipped wedge in the presence of an impressed electric or magnetic line source. The corrugations have the shape of angular sectors. The primary objective of this work is to investigate the impact of corrugations on the scattered field in the shadow region of the structure. An optimally designed corrugated cylinder placed at the tip of a conducting wedge can effectively suppress scattering in the shadow region. Obtained numerical results prove the above concept. Application of an optimally designed corrugated cylinder will be shown for the case of a sectoral horn antenna that is widely used in satellite communications. The aim of this research is to utilize these corrugated tips in horn antenna design for the reduction of side-lobe level and the shaping of the respective E-plane radiation pattern.

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Group-theoretical Methods in Convection Problems

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Basic equations of continuum mechanics reflect deep symmetry properties of space-time and medium, which moves in it. As a corollary, these equations admit a wide Lie group that allows us to apply group-theoretical methods for the study of continuum motion.

As the first application of group-theoretical methods to the convection theory we consider the problem of convective flows in long tubes. Ostroumov (1952) found a class of solutions for Oberbeck-Boussinesq equations, which describe stationary motions in a vertical tube with linear temperature dependence of longitudinal coordinate. It turned out that this solution has a group-theoretical nature. There is a well known solution to Oberbeck-Boussinesq equations describing the two dimensional flow in a horizontal strip under joint action of longitudinal heat flux and transversal gravity field (Birikh, 1966). Asymptotical character of this exact solution was confirmed by experimental and numerical methods (Kirdyashkin, Polezhaev & Fedyushkin, 1983). Its generalization for the case of motion in a cylindrical tube of an arbitrary cross-section was proposed by Pukhnachev (2000). Let z be coordinate along the tube and x, y be coordinates in its cross-section. It is distinguishing that the velocity field in solutions under consideration has three nonzero components depending on x, y and time t only while dependence of temperature θ and pressure on z is linear, and also $\partial\theta/\partial z = \text{const}$. As a result, the original three dimensional problems can be reduced to two dimensional ones. This reduction has a group-theoretical nature: it is provided by the presence of the wide Lie pseudogroup admitted by Oberbeck-Boussinesq equations. Global unique solvability of non-stationary problem and local solvability of stationary problem are established.

The similar results are obtained in the problem of thermal convection in a rotating circular tube with temperature gradient being a linear function of z (Birikh and Pukhnachev, 2011). We underline that both angular velocity of tube and gravity acceleration can be arbitrary functions of t. If the gravity is absent, the latter problem has axially symmetric solution determined from a linear system of equations. Moreover, in this case $\partial\theta/\partial z$ can be function of time and polar radius. This problem admits an effective solution if the longitudinal gradient of pressure is a given function of t. If the liquid flux over the tube cross-section is prescribed, we get an inverse problem, for which the existence and uniqueness theorems are proved. The case of zero flux is of a special interest from the point of view of the solution physical interpretation. A similar inverse problem for the Poiseuille flow was studied by Pileckas & Keblikas (2005). Stationary solutions of this type can be expressed via elementary functions. Besides, we construct solutions of the axially symmetric problems describing the flow of two immiscible liquids with a cylindrical interface. Formally speaking, obtained results deal with motions in infinite tubes. To apply them to description of convection in finite long tubes, it is desirable to have some analogue of the Saint-Venant principle in the classical elasticity theory. We believe that this principle is valid at least for small Rayleigh numbers. It would be interesting to verify our conjecture by numerical experiments.

In general case, properties of liquid depend on temperature. This leads to the problem of group classification for the equations system of thermal gravitational convection. Arbitrary functional elements in these systems are viscosity, heat conductivity coefficients and buoyancy force acceleration as functions of temperature. The problem of group classification was solved by Goncharova (1987), who found the cases of extension of admitted Lie group and constructed a number of exact solutions in these special cases.

Group-theoretical methods are an effective tool for obtaining exact solutions in the theory of thermocapillary (or Marangoni) convection. The corresponding mathematical problem contains an unknown free boundary of flow domain as one of the solution's element. Fortunately, natural free boundary conditions inherit some group properties of the original system of equations. Theorem of free boundary conditions invariance was proved initially by Pukhnachev (1972) for Navier-Stokes equations and generalized by Andreev and Pukhnachev (1983) for Marangoni convection. It should be underlined that these theorems give possibility to construct not only invariant solutions but also partially invariant ones in the sense of Ovsiannikov. Examples of invariant solutions in the coupled thermocapillary and thermogravitational convection demonstrate the motion in a strip with the upper horizontal free surface in the presence of longitudinal temperature gradient (Birikh, 1966) and the Poiseuille-Marangoni flow of two immiscible fluids in an inclined channel (Napolitano, 1980). Pukhnachev (2002) obtained partially invariant solution, which describes deformation of a liquid layer in the weightlessness by thermocapillary forces.

In conclusion, we mention about application of symmetry approach in the branching problem of convective flows. One of them concerns with the selection of modes in the classical Benard-Rayleigh problem and in the Ostroumov problem, which deals with arising of convection in a vertical circular cylinder. An important observation was made by Yudovich (1967). He noted that the systems of branching equations obtained in the Lyapunov-Schmidt procedure inherit a part of symmetry properties of the original equations of convection. This allows reducing the order of the mentioned branching equations. This idea was developed in research of Yudovich's school. In particular, there was considered the Poincare-Andronov-Hopf bifurcation under group invariance conditions (Kolesov and Romanov, 2010).

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The Deformed Exponential Functions of Two Variables – Motivation and Applications

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The development of non-extensive statistical mechanics reinvestigated the adequacy of well-known exponential function. A function was needed which has exponential behavior nearby the origin and behavior of power function in other points [1]. It was leading to various generalizations. Among them, the Tsallis and Kaniadakis exponential functions are the most important [2].

In our papers, we have considered our own deformations of classical exponential function which would have them like special cases with preserved differential and difference properties and sustainable from mathematical point of view. Those functions showed acceptable in explanation of continuous and discrete models of numerous phenomena in the population dynamics and the economy.

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Theoretical Analysis and Numerical Approximation for the Control of Pollutant Transport in Shallow Water

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Analysis of the environmental impact of waste water discharges into aquatic media takes a great importance in the last years. The problem considered here is in the field water-quality improvement by varying the systems, such as flow regulation by means of reservoirs. The criterion functional to be minimized penalizes deviation of fecal coliform distribution from standard value. We address approximate controllability problems for a parabolic equation (evolution of pollutant concentration) associated with Dirac measures. From a practical point of view, a regularized version of the problem is considered. In the first part of this talk we present theoretical analysis of the control of pollutant transport and deal with both 1D and 2D case [2], [3]. We have in these cases convergence results of regularized control problem to the pointwise one. In the second part we present numerical simulation results and various approximation techniques used. The simulation of the transport and fate of compounds within biological systems are done by an hybrid numerical approach combining particle method and finite difference technique. Symmetric TVD scheme [1] for the shallow water equations is provided to minimize numerical diffusion. The technique is a composed method that uses a second order flux in smooth regions but involves some limiting based on the gradient of the solutions so that near discontinuities it reduces to the monotone upwind method. A particle method is proposed to handle the parabolic equation. The difficulty is then to deal with a diffusion term and boundary conditions. In the minimizing algorithm, the gradient of the cost function is evaluated by adjoint techniques and a gradient type method as an iterative solution of the discrete control problem is chosen. The major issue of the numerical part relies on illustrating by direct simulation the effectiveness of this methodology when applied to the control of water quality problem. Relative merits and advantages of this approach are explored.

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Uniform Persistence in Discrete and Continuous Non-autonomous Dynamical Systems with Application to Epidemic Models

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This is an extension of the work of Salceanu and Smith (2009), where boundary attractors for autonomous dynamical systems on the positive orthant of $\mathbb{R}^m$, generated by maps, were characterized as uniformly weak repellers, in order to obtain conditions for uniform persistence. Here we take an unified approach, for both discrete and continuous time non-autonomous systems. We show that when a compact subset of the invariant boundary, that attracts all orbits of the boundary, has certain repelling properties, robust uniform persistence for the complementary dynamics is obtained. We also discuss some particular cases for this boundary attracting set, that often occur in applications, and conclude by giving sufficient conditions for robust uniform persistence of the disease in two epidemic models.

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Energy Transfer in Ratchets Driven by Additive Trichotomous Noise

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The current boom of the ratchet effect, *i.e.*, a directed motion of Brownian particles induced by nonequilibrium fluctuations, with no macroscopic driving applied, in spatially periodic structures has started with Magnasco‘s theoretical work [1]. The initial motivation in this field has come from biology, in particular from the studies of the mechanism of vesicle transport inside eukaryotic cells. Beyond that it was suggested that the ratchet mechanism can be used for obtaining efficient separation methods of nanoscale objects. In our work, the efficiency of the energy transformation of overdamped Brownian particles in a tilted periodic sawtooth potential driven by a nonequilibrium three-level noise and an additive thermal noise is considered analytically. All the physical results discussed have been computed by means of exact formulas. It is established that in a certain parameter region the dependence of the efficiency of energy transformation on noise parameters exhibits a bell-shaped form. Thus, in such parameter regions an increase of the values of noise characteristics (temperature, noise-flatness, correlation time, and noise amplitude) can facilitate the conversion of noise energy into mechanical work. The connection of such a resonance-like behavior of efficiency with the phenomenon of multiple current reversals [2] is also discussed.

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Generating Non-structured Surfaces Defining by n-Degree Polynomials

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A paper presenting a midpoint subdivision of "cubic" Bezier splines was presented in AMiTaNS 2010. A new set of vertices are generated which are the control points for the two sub-segments in the case of curves or the four sub-patches in the case of surfaces. As this method is based on subdivision, after each iteration a larger number of vertices are generated which get closer to the curve/surface. After a number of iterations the new vertices are very close to the actual curve/surface and by displaying them the result is a smooth curve/surface. Unlike most spline methods which are based on regular rectangular patches, the method is generalised to handle surfaces with a non-rectangular grid of vertices.

This paper extents the work to "n-degree" polynomials – other than cubic – which are also generalised to handle surfaces with an arbitrary topology of control vertices.

The Bezier spline segment is given by Eq. (1) where n is the degree of the spline polynomials, V_i are the vertices defining the segment, and $0 \leq u \leq 1$.

$$Q(u) = \sum_{i=0}^{n} V_i B_{i,n}(u) \tag{1}$$

where

$$B_{i,n}(u) = \tbinom{n}{i} u^i (1-u)^{n-i}, \qquad \tbinom{n}{i} = \frac{n!}{i!(n-i)!}.$$

Thus, a segment is controlled by $(n+1)$ vertices in the case of curves, and in the case of surfaces that are based on a regular rectangular grid a patch is controlled by $(n+1) \times (n+1)$ vertices. However, in surfaces with a non-rectangular structure the number of control vertices defining each particular patch depends on the structure surrounding it.

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Model Order Reduction for Industrial Problems

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Cooperation between mathematics and industry is indispensable nowadays in view of the many innovation challenges that industry is faced with. The benefit goes both ways: industry profits from advanced mathematical methods by being able to design higher quality products faster and in a much more optimal way, including all parasitic effects that may be present. Mathematics benefits as a discipline, as the industrial challenges often require the development of new mathematical techniques. In this presentation, we will show examples of such cooperation, specifically in the electronics industry. The field of model order reduction receives much attention in the recent decade, and is important for industry since it allows much faster simulations especially in an optimization context. We will show how dedicated MOR techniques have been developed recently, and how these help to improve the products of the electronics industry.

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Influence of Reservoirs on Pressure Driven Gas Flow in a Micro-channel

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Rapidly emerging micro-electro-mechanical devices create new potential microfluidic applications. A simulation of an internal and external gas flows with accurate boundary conditions for these devices is important for their design. In this paper we study influence of reservoirs used at the micro-channel inlet and outlet on the characteristics of the gas flow in the micro-channel. The problem is solved by using two completely different methods: finite volume method SIMPLE-TS (continuum approach) and Direct Simulation Monte Carlo (molecular approach). We investigate two cases: a short and a relatively long micro-channel. The aspect ratio of the short channel is $A = L_{\mathrm{ch}}/H_{\mathrm{ch}} = 10$, where L_{ch} is the channel length, H_{ch} is the channel height. The long micro-channel has an aspect ration A = 50. Comparisons of results obtained by using both methods for pressure driven flow in a micro-channel with and without reservoirs at the channel ends will be presented. The results were obtained using the grid sites of the South East European Regional Operating Centre.

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On the Invertibility of Classes of Integral Operators via Generalized Factorization

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Wiener-Hopf and Hankel integral equations are associated with a huge amount of applied mathematical problems where they play a central role. In fact, it was already in 1931 that Norbert Wiener and Eberhard Hopf -– when investigating a problem in radiative equilibrium – discovered how to solve a certain class of integral equations which is now classified with their names. After that several generalizations and related types of corresponding equations and operators have been considered in a consequent manner. It is also relevant to mention that since the 1960s and 70s the dominant methods on the study of that type of equations are based on operator theory. In the present talk, we will describe the invertibility property of so-called Wiener-Hopf-Hankel integral operators upon a certain notion of factorization of their Fourier symbols (which shows to be convenient not only for the invertibility but also to the Fredholm property of the operators under study).

The talk is based on a joint work with L. Castro.

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Parameterizing the Shapes of Repelling Quasi-particles of the Vector Nonlinear Schrödinger Equations

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The system of Coupled Nonlinear Schrodinger Equations (SCNLSE) is solved numerically by means of a conservative difference scheme. It was found in previous work [1] that for sufficiently negative values of the cross-modulation coupling parameter, the solitons dynamics were radically different from the classical case in the sense that the solitons repel each other. In the present work we focus on this type of repelling interaction and show that after the collision the shape of the quasi-particle remains qualitatively very similar to the original. We stipulate that the outgoing quasi-particles have the same generic form (a sech envelope modulating a traveling wave), but allow some of the parameters to vary. We employ fitting methods to both real and imaginary parts of the solutions and recover the shape parameters after the collision [2]. We may then characterize them in terms of, for example, changes in carrier frequency or phase speed.

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Approximation of Nonhomogeneous System ODEs with Constant Coefficients by Homogeneous First Order System ODEs with Constant Coefficients

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The general method to solve Nonhomogeneous System ODEs with Constant Coefficients is the method of variation of the constants developed by Lagrange. Sometimes this method leads to integrals that cannot be found exactly. The proposed approximation of systems allows to find the approximate solution using eigenvalues and eigenvectors. The results are closer to the exact solution than the results after the integration by series.

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Fracture Analysis in Functionally Graded Magnetoelectroelastic Plane by the Boundary Integral Equation Method

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A functionally graded magnetoelectroelastic plane with a finite crack is considered. The crack is impermeable and subjected to an anti-plane mechanical and in-plane electric and magnetic load. The fundamental solutions of the coupled system of the governing equations are derived in a closed form by the Radon transform. They are implemented in a non-hypersingular traction boundary integral equation method (BIEM).

A program code in Fortran, based on the BIEM, is created. The numerical examples show the dependence of the stress intensity factor and crack opening displacements on type of the material and the magnitude and direction of the material inhomogeneity in the magnetoelectroelastic medium.

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Optimal Treatment Strategies for Malaria Infection

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We develop a numerical method for estimating optimal parameters in a mathematical model of the within-host dynamics of malaria infection. The model consists of a quasilinear system of partial differential equations. Convergence theory for the computed parameters is provided. Following this analysis, we present several numerical simulations that suggest that periodic treatments that are in synchronization with the periodic bursting rate of infected erythrocytes are the most productive strategies.

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On the Asymptotic Behavior of a Reaction-Diffusion System in a Porous Medium Micro-channel

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The effective behavior of a reaction-diffusion system in a porous medium is analyzed. We deal, at the microscale, with an ε-periodic structure Ω, consisting of two parts: a fluid phase Ω^ε and a solid one, $\Omega \setminus \overline{\Omega}^\varepsilon$; ε represents a small parameter related to the characteristic size of the solid grains.

In such a domain, we analyze the asymptotic behavior, as $\varepsilon \to 0$, of a coupled system of equations, involving diffusion, adsorption and non-smooth chemical reactions. Assuming that the surface of the solid part is physically and chemically heterogeneous and allowing also a surface diffusion modelled by a Laplace-Beltrami operator to take place on this surface, we prove that the effective behavior of our system is governed by a new boundary-value problem, with an additional microvariable and a zero-order extra term proving that memory effects are present in this limit model.

Key words: Homogenization, reactive flows, adsorption, Laplace-Beltrami operator.

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Perturbation Solution for the 2D Shallow-water Waves

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The Boussinesq model of shallow water flow is considered, which contains nonlinearity and fourth-order dispersion. Boussinesq equation has been one of the main soliton models in 1D. To find its 2D solutions, a perturbation series with respect to the small parameter $\epsilon = c^2$ is developed in the present work, where c is the phase speed of the localized wave. Within the order $O(\epsilon^2) = O(c^4)$, a hierarchy is derived consisting of one-dimensional fourth-order equations. The Bessel operators involved are reformulated to facilitate the creation of difference schemes for the ODEs from the hierarchy. The numerical scheme uses a special approximation for the behavioral condition in the singularity point (the origin). The results of this work show that at infinity the 2D wave shape decays algebraically, rather than exponentially as in the 1D cases. The new result can be instrumental for understanding the interaction of 2D Boussinesq solitons, and for creating more efficient numerical algorithms explicitly acknowledging the asymptotic behavior of the solution.

Key words: Boussinesq Equation, two-dimensional solitary waves, perturbation method.

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Radially Projected Finite Elements for Reaction Diffusion Systems

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We develop and analyze two numerical methods to approximate solutions of reaction diffusion systems defined on arbitrary surfaces. In particular, we are interested in reaction diffusion systems that model pattern formation on evolving surfaces. Such systems have numerous applications; examples include patterns on seashells and tropical fish, tumor growth and cell membrane deformation. One of the two methods we propose is based on radially projected finite elements. The power of both of these numerical methods are that they are easy to implement, and all computations are done in logically rectangular coordinates.

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On the Numerical Investigation of Unsteady Solutions for the 2D Boussinesq Paradigm Equation in Moving Frame Coordinate System

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It has been recently shown that the 2D Boussinesq paradigm equation admits stationary translating localized solutions. In order to investigate numerically their time behavior and structural stability, we use a moving frame coordinate system and design a difference scheme with second order truncation error in space and time. The moving frame allows us to keep the localized structure in the center of coordinate system reducing the effects of the reflection from the boundary when the structure approaches one of them. The results from numerical experiments are in good agreement with previous results in fixed coordinates, i.e., for small times the solution preserves its shape, but for larger times it either disperses in the form of decaying ring wave or blows-up.

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On the Conservation Law Forms of the Two- and Three Dimensional Equations of the Nonlinear Elasticity

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The nonlinear finite-deformation elasticity theory has been, and remains, a source of difficult unsolved problems in modern analysis, geometry and computational mathematics. The conservation laws (CLs) such as the conservation of energy, moments and others satisfied by the solutions of differential system play a basic role at the analysis for existence uniqueness and stability of classical and generalized solutions as well as at numerical discretizations.

All CLs of the one-dimensional adiabatic elastodynamics are obtained in [1]. In the present work we look for a full set of CLs forms for basic models equations of the two-and three-dimensional nonlinear elasticity.

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A Third-order Scheme to Guarantee Non-negative Coefficients for Advection-diffusion Equations

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According to Godunov theorem for numerical calculations of advection equations, there exist no higher-order schemes with constant positive difference coefficients in a family of polynomial schemes with an accuracy exceeding the first-order. In case of advection-diffusion equations, so far there have been not found stable schemes with positive difference coefficients in a family of numerical schemes exceeding the second-order accuracy. We propose a third-order computational scheme for numerical fluxes to guarantee the non-negative difference coefficients of resulting finite difference equations for advection-diffusion equations. The present scheme is optimized so as to minimize truncation errors while fulfilling the positivity condition of the difference coefficients which are variable depending on the local Courant number and diffusion number. The feature of the present optimized scheme consists in keeping the third-order accuracy anywhere without any numerical flux limiter by using the same stencil number as conventional third-order schemes such as KAWAMURA and UTOPIA schemes. We extend the present method into multi-dimensional equations. Numerical experiments for linear and nonlinear advection-diffusion equations were performed and the present scheme's applicability to nonlinear Burger's equation was confirmed.

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Digital Pulse Processing of Nuclear Detectors

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Digital signal processing algorithms for nuclear particle spectroscopy are described along with a digital pile-up elimination method applicable to equidistantly sampled detector signals pre-processed by a charge-sensitive preamplifier. The signal processing algorithms are provided as recursive procedures which can be easily programmed using modern computer programming languages. The influence of the number of bits of the sampling analogue-to-digital converter to the final signal-to-noise ratio of the spectrometer is considered. The pile-up elimination method was originally developed for fission fragment spectroscopy using a Frisch-grid back-to-back double ionisation chamber and was mainly intended for pile-up elimination in case of high alpha-radioactivity of the fissile target. The influence of the pile-up elimination scheme on the final resolution of the spectrometer is investigated in terms of the distance between piled-up pulses using high a purity germanium detector. The efficiency of the developed algorithms is compared with other signal processing schemes published in literature.

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Total number of abstracts 92